New developments in chaos game theory

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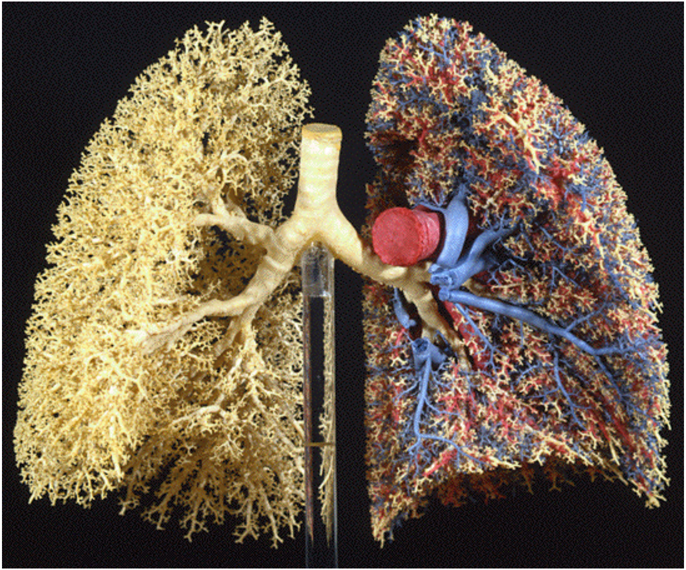
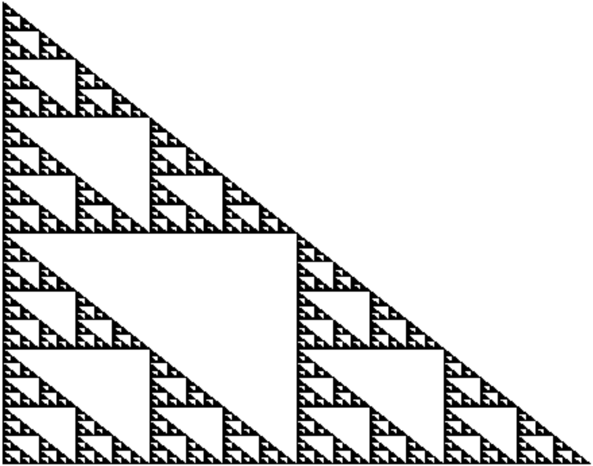
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

Fractals

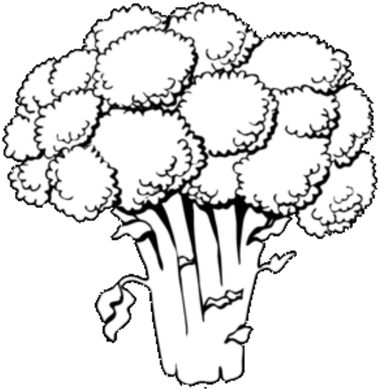


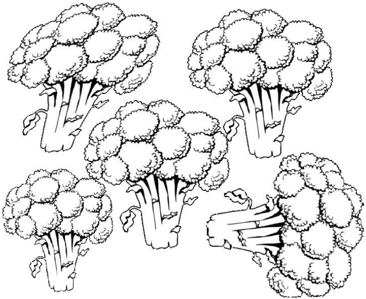
Fractals

Fractals



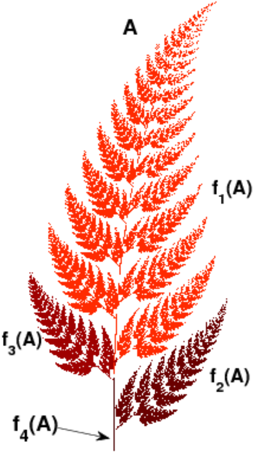
Selfsimilarity





$$A = f_1(A) \cup f_2(A) \cup \cdots \cup f_N(A)$$

Selfsimilarity



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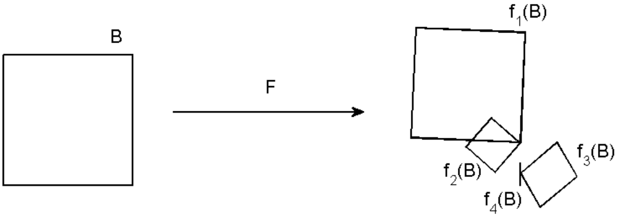
Selfsimilar sets consist of their copies.

Hutchinson operator



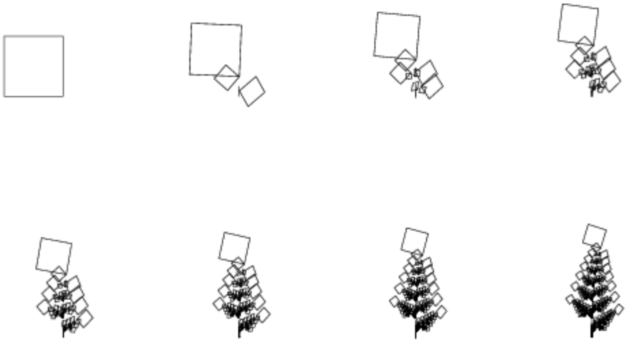
$$F(B) = f_1(B) \cup f_2(B) \cup \cdots \cup f_N(B)$$

Hutchinson operator

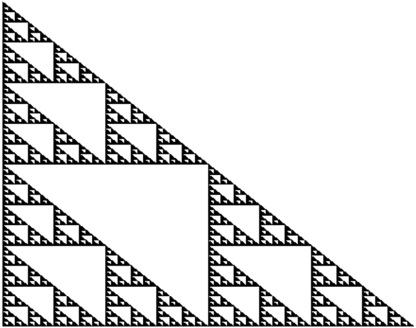
$$F(B) = f_1(B) \cup f_2(B) \cup \cdots \cup f_N(B)$$

If f_i are contractions

$$F^n(B) \to A$$



Sierpinski triangle



$$f_1(x) = \frac{x + a_1}{2}$$

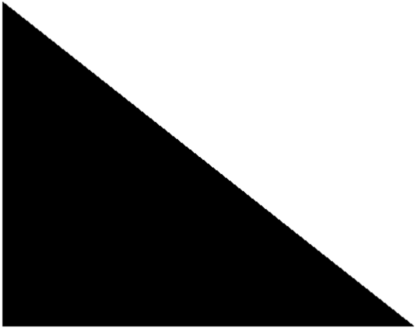
$$f_2(x) = \frac{x + a_2}{2}$$

$$f_3(x) = \frac{x + a_3}{2}$$

$$a_1 = [0, 0], a_2 = [1, 0],$$

$$a_3 = [0, 1]$$

Sierpinski triangle



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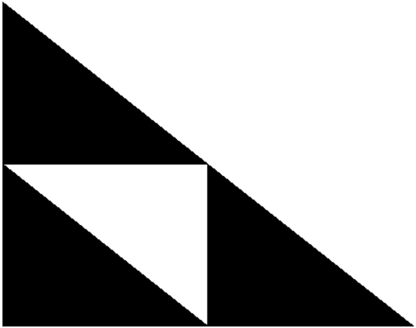

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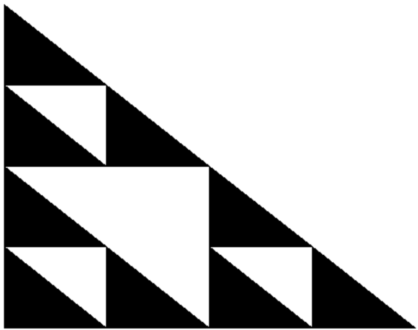

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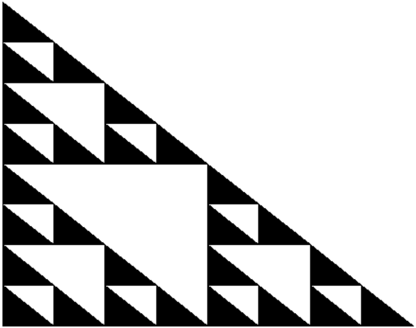

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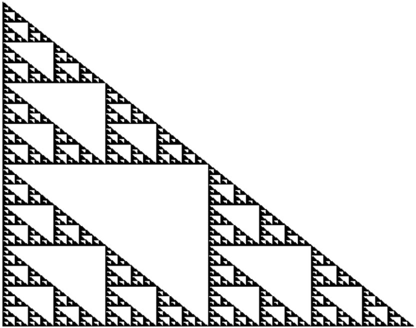

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Chaos game

Definition

Given contractive maps with probabilities $\{f_i, p_i, i = 1, 2, \ldots, m\}$, we construct a sequence

$$\{x_i\}_{i=1}^n,$$

where

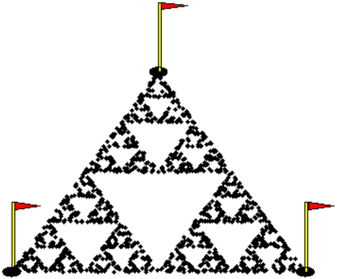
$$x_{i+1} = f_{u_i}(x_i),\ u_i \in \{1, 2, \ldots, m\}.$$

Mappings f_j are taken with given probability $P(u_i = j) = p_j,\ j \in \{1, 2, \ldots, m\}$.

Theorem

The sequence $\{x_i\}_{i=1}^\infty$ *approximates the fractal with probability one.*

Chaos game



$$f_1(x) = \frac{x + a_1}{2}$$

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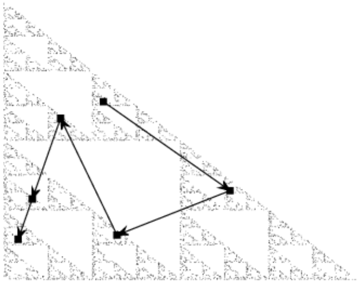
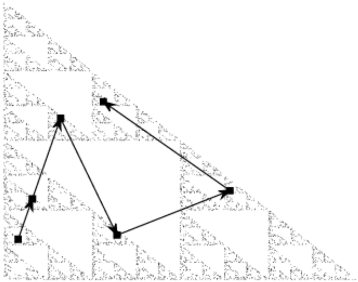
Chaos game

stochastic

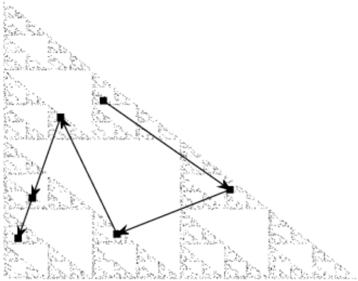


... and chaotic as well



Chaos game

stochastic



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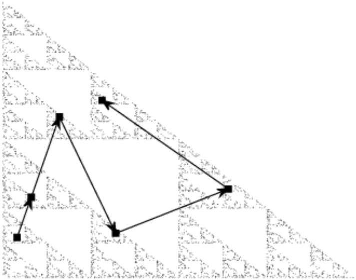
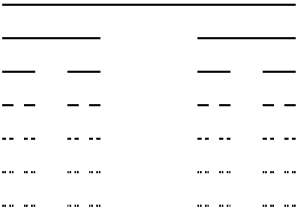


taken backwards

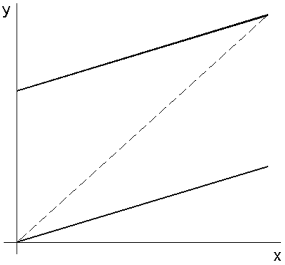
Cantor set

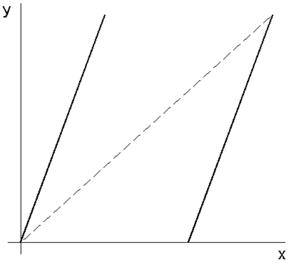


$$f_1(x) = \frac{x}{3}$$

$$f_2(x) = \frac{x+2}{3}$$

Chaos game on the Cantor set





chaotic map

Use of the chaos game

- Visualization of fractals
- image processing
- Analogy of Monte Carlo method for fractal measures

References

Michael F. Barnsley, *Fractals everywhere,* Academic Press, New York (2003).

What happens if the maps are not contractive?

Michael F. Barnsley, Krzysztof Lesniak and Miroslav Rypka, *Chaos game for IFSs on topological spaces,* arXiv preprint arXiv:1410.3962 (2014).

Thank you for your attention